Linear Algebra Practice Problems, Part II

1. (This is one of the oldest known linear algebra problem, which was found on a Babylonian clay tablet from around 300 BC. This is classic ag econ!)

There are two fields whose total area is 1800 square yards. One produces grain at the rate of ​ of a bushel per square yard while the other produces grain at the rate of ​ a bushel per square yard. If the total yield is 1100 bushels, what is the size of each field?

1. Use Cramer’s Rule to solve these systems:
2. Solve for z using Cramer’s Rule:
3. Show that a matrix with two identical rows must have a determinant of zero, using only the property that interchanging two rows swaps the sign of the determinant.
4. For each of the three types of elementary matrices (row switching, row multiplication, row addition), find the inverse and the determinant.
5. Expand the product xx’ where x is a vector in . What is the rank of the resulting matrix?
6. Prove that the eigenvalues of a positive definite matrix are all strictly positive. (hint: consider the quadratic form x’Ax using the spectral decomposition of A and the unit eigenvectors (eigenvectors normalized to have length 1) as x.